Dynamics of Shearfree Dissipative Collapse in $f(G)$ Gravity

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Abstract

In this paper, we study the dynamics of shearfree dissipative gravitational collapse in modified Gauss-Bonnet theory of gravity - so called $f(G)$ gravity. The field equations of $f(G)$ gravity are applied to shearfree spherical interior geometry of the dissipative star. We formulate the dynamical as well as transport equations and then couple them to investigate the process of collapse. We conclude that the gravitational force in this theory is much stronger as compared to general relativity which indicates increase in the rate of collapse. Further, the relation between the Weyl tensor and matter components is established. This shows that for constant $f(G)$ the vanishing of the Weyl scalar leads to the homogeneity in energy density and vice versa.

1   Introduction

Recently, a lot of work has been done on alternative theories of gravity for the identification of dark energy (DE) which is said to be responsible for

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accelerated expansion of the universe.[1] These theories are attractive due to their consistency with the astrophysical observations and local gravitational experiments.[2] The simplest generalization to general relativity (GR) is $f(R)$ gravity in which f is an arbitrary function of the Ricci scalar R. Although, it is the simplest modification of GR but it is not possible generally to formulate such $f(R)$ model which is consistent with the solar system tests. Many conditions for the validity of $f(R)$ models have been proposed.[3]

Another modified theory of gravity "the modified Gauss-Bonnet theory" was proposed by several authors.[4]−[6] In this theory, GR is modified by introducing some arbitrary function $f(G)$ in Einstein-Hilbert action, where $G = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\gamma\delta}R_{\mu\nu\gamma\delta}$ is the Gauss-Bonnet invariant. This modification (consistent with observational constraints) is related to string-inspired dilaton theory.[7] Cognola et al.[8] found that such model can produce transition form matter dominated era to accelerated phase.

The dynamics of the dissipative shearfree gravitational collapse is an important issue. In GR, this problem was initially formulated many years ago by Misner and Sharp[9] for adiabatic collapse and by Misner[10] for non-adiabatic collapse. The dissipative process during the collapse of massive star is important due to the energy loss during formation of neutron star or black hole.[11] It is shown[12],[13] that gravitational collapse is a dissipative process, so the effects of the dissipation must be included in the study of collapse. Herrera and Santos[14] discussed the dynamics of shearfree anisotropic radiating fluid. Chan[15] studied the realistic model of radiating star with shear viscosity.

Herrera[16] explored the inertia of heat and its role in the dynamics of dissipative collapse. Herrera et al.[17] also derived the dynamical equations by including dissipation in the form of heat flow, radiation, shear and bulk viscosity and then coupled with causal transport equation. They investigated that shearfree condition for quasi-static (slowly evolving) non-dissipative systems in Newtonian limit leads to homologous collapse. Di Prisco et al.[18] investigated the dynamics of spherically symmetric charged viscous non-adiabatic gravitational collapse. Sharif and his collaborators [19]−[25] have discussed the dynamics of charged and uncharged viscous dissipative gravitational collapse in GR as well as in $f(R)$ gravity.

In this paper, we discuss the dynamics of the dissipative shearfree gravitational collapse in $f(G)$ gravity. We use the field equations of $f(G)$ gravity[26] developed by using covariant gauge invariant (CGI) perturbation approach with $3 + 1$ formalism. The collapsing matter is taken as dissipative isotropic

fluid bounded by the shearfree interior geometry. This spacetime is matched with the Vaidya exterior solution by using Darmois junction conditions [27]. The plan of the paper is as follows: In the next section, we describe the field equations in $f(G)$ gravity and matching conditions. We formulate the dynamical as well as transport equations and then couple them in section **3**. In section **4**, we discuss the relation between the Weyl tensor and matter variables. The last section provides discussion of the results.

2 Field Equations in Modified Gauss-Bonnet Gravity

The Einstein-Hilbert action for the modified Gauss-Bonnet gravity is

$$S = \int d^4x \sqrt{-g} \left(\frac{R + f(G)}{2\kappa} + \mathcal{L}_m \right), \tag{1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, R is the Ricci scalar, $f(G)$ is an arbitrary function of Gauss Bonnet invariant G, $\mathcal{L}_m$ is the matter Lagrangian and κ is coupling constant. The variation of this action with respect to the metric tensor gives the field equations

$$
\begin{aligned}
G_{\mu\nu} =\ & \kappa T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} f - 2FRR_{\mu\nu} + 4FR_\mu^\gamma R_{\nu\gamma} - 2FR_{\mu\gamma\delta\omega}R_\nu^{\gamma\delta\omega} \\
& - 4FR_{\mu\gamma\delta\nu}R^{\gamma\delta} + 2R\nabla_\mu\nabla_\nu F - 2g_{\mu\nu}\nabla^2 F - 4R_\mu^\gamma\nabla_\nu\nabla\gamma F \\
& - 4R_\nu^\gamma\nabla_\mu\nabla\gamma F + 4R_{\mu\nu}\nabla^2 F + 4g_{\mu\nu}R^{\gamma\delta}\nabla_\gamma\nabla_\delta F - 4R_{\mu\gamma\nu\delta}\nabla^\gamma\nabla^\delta F,
\end{aligned} \tag{2}
$$

where $F = \partial f(G)/\partial G$. The energy-momentum tensor for dissipative fluid is

$$T_{\mu\nu} = (\rho + p)V_\mu V_\nu - pg_{\mu\nu} + q_\mu V_\nu + q_\nu V_\mu, \tag{3}$$

where ρ, p, V_μ and q_μ are density, pressure, four velocity and radial heat flux, respectively.

In general, it is very difficult to handle the field equations. We use a simplified form of these equations derived by Li et al.[26] through CGI perturbation using $3 + 1$ formalism and the effective energy-momentum approach. Since there are nonlinear terms in $R_{\mu\nu}$ and $R_{\mu\nu\lambda\sigma}$ which are expressed in terms of dynamical quantities, so the field equations and Gauss-Bonnet invariant are simplified as follows

$$G_{\mu\nu} = \kappa(T_{\mu\nu} + T_{\mu\nu}^G), \tag{4}$$

where $T^G_{\mu\nu}$ is the Gauss-Bonnet correction term. Using CGI approach, the components of $T^G_{\mu\nu}$ are evaluated in terms of the dynamical quantities as[26]

$$\rho^G = \frac{1}{\kappa}\left(\frac{1}{2}(f - FG) + \frac{2}{3}(\Omega - 3\Psi)(\dot{F}\Theta + \tilde{\nabla}^2 F)\right), \qquad (5)$$

$$-p^G = \frac{1}{\kappa}\left(\frac{1}{2}(f - FG) + \frac{2}{3}(\Omega - 3\Psi)\ddot{F} - \frac{8}{9}\Omega(\Theta\dot{F} + \tilde{\nabla}^2 F)\right), \qquad (6)$$

$$q^G_\mu = \frac{1}{2}\left(-\frac{2}{3}(\Omega - 3\Psi)(\tilde{\nabla}_\mu\dot{F} - \frac{1}{3}\Theta\tilde{\nabla}_\mu F) + \frac{4}{3}\dot{F}\Theta\zeta_\mu\right). \qquad (7)$$

Here, $\tilde{\nabla}$ is spatial covariant derivative, $\Theta = V^\mu_{;\mu}$ is expansion scalar and the quantities Ω, Ψ, ζ_μ are expressed in terms of dynamical quantities as follows

$$\Omega = -(\dot{\Theta} + \frac{1}{3}\Theta^2 - \tilde{\nabla}^\mu A_\mu), \qquad (8)$$

$$\Psi = -\frac{1}{3}(\dot{\Theta} + \Theta^2 + \tilde{R} - \tilde{\nabla}^\mu A_\mu), \qquad (9)$$

$$\zeta_\mu = -\frac{2\tilde{\nabla}_\mu\Theta}{3} + \tilde{\nabla}^\nu\sigma_{\mu\nu} + \tilde{\nabla}^\nu\omega_{\mu\nu}, \qquad (10)$$

where $\tilde{R}$ is the Ricci scalar of $3D$ spatial spherical surface, $\sigma_{\mu\nu} = V_{(\mu;\nu)} - \dot{V}_{(\mu}V_{\nu)} - \frac{1}{3}\Theta h_{\mu\nu}$ (where $h_{\mu\nu} = g_{\mu\nu} - V_\mu V_\nu$) is the shear tensor, $A_\mu = V_{\mu;\nu}V^\nu$ is four acceleration and $\omega_{\mu\nu} = V_{[\mu;\nu]} + \dot{V}_{[\mu}V_{\nu]}$ is the vorticity tensor. The Gauss-Bonnet invariant G in CGI approach is

$$G = 2(\frac{1}{3}R^2 - R^{\mu\nu}R_{\mu\nu}), \qquad (11)$$

where

$$R = -2\dot{\Theta} - \frac{4}{3}\Theta^2 + 2\tilde{\nabla}^\mu A_\mu - \tilde{R},$$

$$R^{\mu\nu}R_{\mu\nu} = \frac{4}{3}(\dot{\Theta}^2 + \dot{\Theta}\Theta^2 + \frac{1}{3}\theta^4) + \frac{2}{3}(\dot{\Theta} + \Theta^2)\tilde{R} - \frac{8}{3}(\dot{\Theta} + \frac{1}{2}\Theta^2)\tilde{\nabla}^\mu A_\mu.$$

Now we assume that the fluid (3) is bounded by the spherical boundary Σ. The metric interior to Σ is assumed to be comoving and shearfree in the following form

$$ds^2 = X^2 dt^2 - Y^2(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2), \qquad (12)$$

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where X and Y are functions of t and r. The corresponding four velocity and heat flux take the form

$$V^\mu = X^{-1}\delta^\mu{}_0, \quad q^\mu = q\delta^\mu{}_1, \tag{13}$$

where $V^\mu q_\mu = 0$. The expansion scalar for the fluid sphere is $\Theta = \frac{3\dot{Y}}{YX}$. Using Eqs.(3), (4), (12) and (13), the field equations yield

$$8\pi(\rho + \rho^G)X^2 = 3\left(\frac{\dot{Y}}{Y}\right)^2 - \left(\frac{X}{Y}\right)^2\left(2\frac{Y''}{Y} + \left(\frac{Y'}{Y}\right)^2 + \frac{4Y'}{Yr}\right), \tag{14}$$

$$8\pi(q + q^G)XY^2 = 2\left(\frac{\dot{Y}'}{Y} - \frac{\dot{Y}Y'}{Y^2} - \frac{\dot{Y}X'}{YX}\right), \tag{15}$$

$$8\pi(p + p^G)Y^2 = \left(\frac{Y'}{Y}\right)^2 + \frac{2}{r}\left(\frac{Y'}{Y} + \frac{X'}{X}\right) + 2\frac{X'Y'}{XY} - \left(\frac{Y}{X}\right)^2$$

$$\times \left(2\frac{\ddot{Y}}{Y} + \left(\frac{\dot{Y}}{Y}\right)^2 - 2\frac{\dot{X}\dot{Y}}{XY}\right), \tag{16}$$

$$8\pi(p + p^G)rY^2 = r^2\left(\frac{Y''}{Y} - \left(\frac{Y'}{Y}\right)^2 + \frac{1}{r}\left(\frac{Y'}{Y} + \frac{X'}{X}\right) + \frac{X''}{X}\right)$$

$$\times \left(2\frac{\ddot{Y}}{Y} + \left(\frac{\dot{Y}}{Y}\right)^2 - 2\frac{\dot{X}\dot{Y}}{XY}\right), \tag{17}$$

where ρ^G, p^G and q^G correspond to Gauss-Bonnet contribution to GR. Here dot and prime indicate derivative with respect to time and radial coordinate, respectively. Making use of Eqs.(5)-(7), these turn out to be

$$\kappa\rho^G = \frac{1}{2}(f - GF) + \frac{1}{3Y^2}\left(\frac{6\dot{Y}}{X^2} + \left(\frac{Y'}{Y}\right)^2 + \frac{4Y'}{Yr} + 2\frac{Y''}{Y}\right)$$

$$\times \left(\frac{3\dot{F}\dot{Y}}{XY} - \tilde{\nabla}^2 F\right), \tag{18}$$

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$$
\kappa p^G = \frac{1}{2}(GF - f) - \frac{1}{3Y^2}\left(\frac{6\dot{Y}}{X^2} + \left(\frac{Y'}{Y}\right)^2 + \frac{4Y'}{Yr} + 2\frac{Y''}{Y}\right)\ddot{F}
$$

$$
- \frac{8}{3}\left(\frac{\ddot{Y}}{XY} - \frac{\dot{X}}{Y} - \frac{\dot{Y}}{XY} + \frac{\dot{Y}^2}{X^2Y^2} - \frac{1}{3Y^2}\left(\frac{X''}{X} - \frac{X'^2}{X} + \frac{X'}{rX}\right)\right)
$$

$$
\times \left(\frac{3\dot{F}\dot{Y}}{XY} - \tilde{\nabla}^2 F\right), \tag{19}
$$

$$
\kappa q^G = \frac{4}{3Y^2}\left(\frac{6\dot{Y}}{X^2} + \left(\frac{Y'}{Y}\right)^2 + \frac{4Y'}{Yr} + 2\frac{Y''}{Y}\right)\left(\frac{\dot{Y}F'}{YX} - \dot{F}'\right)
$$

$$
+ \frac{8\dot{F}\dot{X}}{YX}\left(\frac{\dot{Y}Y'}{XY^2} + \frac{\dot{Y}X'}{X^2Y} - \frac{\dot{Y}'}{XY}\right). \tag{20}
$$

The Gauss-Bonnet invariant is found from Eq.(11) as

$$
G = \frac{2}{3}\left(-6\left(\frac{\dot{Y}}{XY}\right)^{\cdot} - 12\frac{\dot{Y}^2}{(XY)^2} - \frac{2}{Y^2}\left(\frac{X'Y'}{XY} - \left(\frac{X'}{X}\right)'\right)\right)
$$

$$
+ 4\frac{X'(Yr)'}{XY^3r} - 2\left(\frac{4Y'}{Y^3r} + \frac{Y''}{Y^3} - \frac{Y'^2}{Y^4r}\right)\right)^2 - 8\left(\left(\frac{\dot{Y}}{(XY)}\right)^{\cdot}\right)^2
$$

$$
+ \left(\frac{3\dot{Y}}{XY}\right)^2\left(\left(\frac{3\dot{Y}}{XY}\right)^{\cdot} + \frac{1}{3}\left(\frac{3\dot{Y}}{XY}\right)^2\right) + \frac{2}{3}\left(\left(\frac{3\dot{Y}}{XY}\right)^{\cdot} + \frac{1}{2}\left(\frac{3\dot{Y}}{XY}\right)^2\right)
$$

$$
\times \left(\frac{4Y'}{Y^3r} + \frac{Y''}{Y^3} - \frac{Y'^2}{Y^4r} + \frac{4}{Y^2}\left(\frac{X'Y'}{XY} - \left(\frac{X}{X}\right)'\right)\right) - \frac{X'Y'r}{XY^3r}\right). \tag{21}
$$

It follows from Eq.(15) that

$$
8\pi(q + q^G)Y^2 = \frac{2\Theta'}{3}. \tag{22}
$$

If $q + q^G = 0$, then $\Theta' = 0$ and we have the condition for homogenous collapse. The Misner-Sharp mass becomes[9]

$$
m(r,t) = \frac{r^3}{2}\left(\frac{Y\dot{Y}^2}{X^2} - \frac{Y'^2}{Y} - \frac{2Y'}{r}\right). \tag{23}
$$

We assume that the exterior region to boundary surface Σ is described by the Vaidya spacetime which represents the geometry of the radiating star given by

$$ds^2 = (1 - \frac{M(\nu)}{\hat{R}})d\nu^2 + 2d\nu d\hat{R} - \hat{R}^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (24)$$

Applying the same procedure as discussed in[19)−25)], we find that the continuity of the first and second fundamental forms over the boundary surface Σ gives the continuity of the gravitational masses across Σ (*i.e.*, $m(r,t) = M(\nu)$) and $p + p^G \overset{\Sigma}{=} (q + q^G)Y$. This implies that the effective pressure is balanced by the effective radial heat flux on the boundary.

3   Dynamical and Transport Equations

In order to study the dynamics of the field equations, we use the Misner-Sharp formalism[9)]. Here velocity of the collapsing fluid is given by

$$U = rD_tY, \qquad (25)$$

where $U < 0$ for radially inward motion of the fluid and $D_t = \frac{1}{X}\frac{\partial}{\partial t}$. From the definition of the mass function (23), we can write

$$\frac{(Yr)'}{Y} = \left(1 + U^2 + \frac{2m(r,t)}{rY}\right)^{\frac{1}{2}} = E. \qquad (26)$$

This is known as the energy of the collapsing fluid. The proper time derivative of the mass function (23) leads to

$$D_tm = r^3\left(\frac{Y\dot{Y}\ddot{Y}}{X^3} + \frac{1}{2}\left(\frac{\dot{Y}}{X}\right)^3 - \frac{Y\dot{Y}^2\dot{X}}{X^4} + \frac{1}{2}\frac{\dot{Y}Y'^2}{XY^2} - r^3\frac{Y'\dot{Y}'}{YX} - \frac{\dot{Y}'}{rX}\right). \qquad (27)$$

Using Eqs.(15), (16), (24) and (25), this can be written as

$$D_tm = -4\pi\left[(p + q^G)U + (q + q^G)YE\right](rY)^2 \qquad (28)$$

which give the variation of the fluid energy inside a sphere of radius rY. Since $U < 0$, so the first term on right side of this equation increases the energy of the system while the negative sign with second term implies the leaving of energy from the system.

We also define proper radial derivative $D_R = \frac{1}{R'}\frac{\partial}{\partial r}$, where $R = rY$ is the proper areal radius of the sphere. The proper radial derivative of Eq.(23) leads to

$$
\begin{aligned}
D_R m \;=\;& \frac{Y}{(rY)'}\left(-r^3\frac{Y'Y''}{Y^2} - r^2\frac{Y''}{Y} + \frac{r^3}{2}\left(\frac{Y'}{Y}\right)^2 - \frac{3r^2}{2}\left(\frac{Y'}{Y}\right)^2\right. \\
& \left. -\;2r\frac{Y'}{Y} - r^3\frac{X'\dot{Y}^2}{X^3} + \frac{r^3}{2}\frac{Y'\dot{Y}^2}{X^2Y} + r^3\frac{\dot{Y}'\dot{Y}}{X^2} + \frac{3r^2}{2}\left(\frac{\dot{Y}}{X}\right)^2\right). \quad (29)
\end{aligned}
$$

Using Eqs.(14), (15), (24) and (25) in (29), it follows that

$$
D_R m = 4\pi\left((\rho + \rho^G) + (q + q^G)Y\frac{U}{E}\right)(rY)^2. \quad (30)
$$

This equation describes the variation in energy between two adjacent layers of the fluid inside the spherical boundary. The Gauss-Bonnet term effects the density and heat flux. Since heat flux is directed outward due to $U < 0$, so the Gauss-Bonnet term causes the system to radiate away effectively.

The acceleration of the infalling matter inside the spherical boundary is obtained by using Eqs.(16), (23) and (25) as

$$
D_t U = -\left(\frac{m}{(rY)^2} + 4\pi(p + q^G)(rY)\right) + \frac{X'(rY)'}{XY^2}. \quad (31)
$$

The r component of the energy-momentum conservation, $(T^{1\beta} + T^{G1\beta})_{;\beta} = 0$, implies that

$$
\frac{1}{Y}(p + q^G)' + \frac{X'}{YX}(\rho + \rho^G + p + q^G) + (\dot{q} + \dot{q}^G)\frac{Y}{X} + 5(q + q^G)\frac{\dot{Y}}{X} = 0. \quad (32)
$$

Using the value of $\frac{X'}{X}$ from this equation in Eq.(31) with (23) and (25), we obtain the dynamical equation

$$
(\rho + \rho^G + p + q^G)D_t U = -(\rho + \rho^G + p + q^G)\left[m + 4\pi(p + q^G)R^3\right]\frac{1}{R^2}
$$
$$
-\;E^2\left[D_R(p + q^G)\right] - E[5Y(q + q^G)\frac{U}{R} + YD_t(q + q^G)]. \quad (33)
$$

This equation has the "Newtonian" form, i.e., $force = mass\ density \times acceleration$, where mass density $= \rho + \rho^G + p + q^G$. This implies that

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Gauss-Bonnet term effects the mass density due to higher curvature. The first square brackets on right side is the gravitational force, whose Newtonian part is m and relativistic part is $p+q^G$, hence the Gauss-Bonnet term effects gravitational force. The second term is the hydrodynamical force, it resists against collapse because $D_R(p + q^G) < 0$. The last square brackets give the contribution of heat flux to the dynamics of the collapsing system. Here the first term is positive ($U < 0$, $(q + q^G) > 0$), implying that outflow of heat flux reduces the rate of collapse by producing radiations in the exterior of the collapsing sphere. The effects of $D_t(q + q^G)$ will be explained below by introducing the heat transport equation.

Now we discuss the transportation of heat during the shearfree collapse of radiative fluid in $f(G)$ gravity. For this purpose, we use the equation derived from Muller-Israel-Stewart phenomenological theory for non-adiabatic fluids [28),29)]. It is known that Maxwell-Fourier law[30)] for the heat flux leads to diffusion equation which indicates perturbation at very high speed. For the relativistic non-adiabatic fluids, this is given by Eckart[31)] and Landau.[32)] To resolve this problem, many relativistic theories[28)−30)] have been proposed. The common point of all these theories is their validity for non-vanishing relaxation time. These theories provide the heat transport equation which is hyperbolic equation.

The transport equation for the heat flux in this case reads[14)]

$$\tau h^{\mu\nu} V^\lambda \tilde{q}_{\nu;\lambda} + \tilde{q}^\mu = -\kappa h^{\mu\nu}(T_{,\nu} + TA_\nu) + \frac{1}{2}\kappa T^2 \left(\frac{\tau V^\nu}{\kappa T^2}\right)_{;\nu} \tilde{q}^\mu, \qquad (34)$$

where $h^{\mu\nu}$ is the projection tensor, $\tilde{q} = q + q^G$, τ is relaxation time, T is temperature and κ is thermal conductivity. For the interior spacetime, this equation reduces to

$$\tau \frac{\partial}{\partial t}[(q + q^G)Y] + (q + q^G)XY^2 = -\kappa(TA)' - \frac{\kappa T^2(q + q^G)Y^2}{2}\left(\frac{\tau}{\kappa T^2}\right)$$
$$- 3\frac{\tau \dot{Y}Y(q + q^G)}{2}. \qquad (35)$$

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Using Eqs.(25) and (30), this implies

$$YD_t(q+q^G) = -\kappa T\frac{D_t U}{\tau E} - \frac{\kappa \acute{T}}{\tau Y} - \frac{Y(q+q^G)}{\tau}\left(1+\frac{\tau U}{rY}\right)$$

$$- \frac{\kappa T}{\tau E}[m+4\pi(p+q^G)R^3]R^{-2} - \frac{\kappa T^2(q+q^G)Y}{2XE}\frac{\partial}{\partial t}\left(\frac{\tau}{\kappa T^2}\right)$$

$$- \frac{3UY(q+q^G)}{2R}. \tag{36}$$

In order to see the effects of heat flux on the dynamics of collapsing sphere in the modified Gauss-Bonnet gravity, we couple the dynamical and heat transport equations given by Eqs.(33) and (36). This coupling leads to

$$(\rho+\rho^G+p+q^G)(1-\alpha)D_t U = F_{grav}(1-\alpha) + F_{hyd} + \frac{E\kappa T'}{\tau Y}$$

$$+ \frac{E(q+q^G)Y}{\tau} - \frac{4E(q+q^G)YU}{R} + \frac{\kappa T^2(q+q^G)Y}{2XE}\frac{\partial}{\partial t}\left(\frac{\tau}{\kappa T^2}\right)$$

$$+ \frac{3UY(q+q^G)}{2R}. \tag{37}$$

Here

$$F_{grav} = -(\rho+\rho^G+p+q^G)\left(m+4\pi(p+q^G)R^3\right)\frac{1}{R^2},$$

$$F_{hyd} = -E^2[D_R(p+q^G)],$$

$$\alpha = \frac{\kappa T}{\tau(\rho+\rho^G+p+q^G)}.$$

Now we analyze the effects of the Gauss-Bonnet term on the dynamics of the collapsing radiative fluid. Since the presence of Gauss-Bonnet term in α (thermal coefficient) effects the value of α and the factor $(1-\alpha)$ being the the multiple of F_{grav} would effects the value of F_{grav}. In the definition of α, κ, τ and T (as mentioned after Eq.(34)) are positive for a real dissipative gravitating source [14] and $(\rho+p)>0$, for collapsing fluid to satisfy the weak energy condition. For ρ^G and q^G, we have two cases

- When $\rho^G>0$ and $q^G>0$, then value of α in this case will be less as compared to GR and value of $(1-\alpha)$ will be larger (as compared to GR) and consequently gravitational force will be stronger as compared to GR and rate of collapse will be increased. For example if $f(G)=f_0$, a positive constant, Eqs.(18)-(20) yield $\rho^G=-p^G=\frac{f_0}{2}$ and $q^G=0$ and we have $\rho^G=\frac{f_0}{2}>0$ and rate of collapse will be increased.

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- When $\rho^G < 0$ and $q^G < 0$, then value of α in this case will be larger as compared to GR and value of $(1 - \alpha)$ will be less (as compared to GR) and consequently gravitational force will be weaker as compared to GR and rate of collapse will be decreased. For example if $f(G) = f_0 < 0$ in above case, then we have $\rho^G = \frac{f_0}{2} < 0$ and rate of collapse will be decreased. This can be verified for the more general $f(G)$ models.

If α approaches to a critical value of 1 then the inertial and gravitational forces of the system become zero and the system will be in hydrostatic equilibrium. Notice that α appears in the system due to its thermal conductivity and temperature. The value of temperature for which $\alpha \to 1$ is equivalent to the expected amount of temperature that might be reached during the radiative collapse in supernova explosion [33]. Thus α increases during the supernova explosion if the relaxation time $\tau \neq 0$. When α crosses the critical value, the gravitational force plays the role of antigravity and expansion would occur in the system.

4 Relation Between Weyl Tensor and Matter Variables

In this section, we establish a relation between the Weyl tensor and density inhomogeneity which helps to extract some information about the gravitational arrow of time. The Weyl tensor leads to the Weyl scalar $\mathcal{C}^2 = C^{\mu\nu\lambda\sigma} C_{\mu\nu\lambda\sigma}$ which is given by

$$\mathcal{C}^2 = \mathcal{R} - 2R^{\alpha\beta} R_{\alpha\beta} + \frac{1}{3} \mathrm{R}^2, \tag{38}$$

where $\mathcal{R} = R^{\mu\nu\lambda\sigma} R_{\mu\nu\lambda\sigma}$ is the Kretchman scalar. Using the field equations in Eq.(46) in appendix, Eq.(38) yields

$$\epsilon = m - \frac{4\pi}{3} R^2 (\rho + \rho^G), \tag{39}$$

where

$$\epsilon = \frac{\mathcal{C}}{\sqrt{48}} R^3. \tag{40}$$

From Eqs.(30) and (39), it follows that

$$D_R \epsilon = 4\pi R^2 \left[(q + q^G) Y \frac{U}{E} - D_R(\rho + \rho^G) \frac{R}{3} \right]. \tag{41}$$

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For $f(G) = f_0$, a constant, Eqs.(18)-(20) yield $\rho^G = -p^G = \frac{f_0}{2}$ and $q^G = 0$, which implies that for constant $f(G)$, we have ΛCDM, model. Assuming that the fluid is non-dissipative, Eq.(41) yields

$$D_R \epsilon + \frac{4\pi}{3} R^3 D_R(\rho + \frac{f_0}{2}) = 0. \qquad (42)$$

If we take $D_R \epsilon = D_R \left(\frac{\mathcal{C}}{\sqrt{48}} R^3 \right) = 0$, using regular axis condition, i.e., $R \neq 0$, we have $\mathcal{C} = 0$. Thus Eq.(42) implies that $D_R \rho = 0$. This means that the conformal flatness condition produces homogeneity in the energy density and vice versa. Equation (42) gives the relationship between the energy density inhomogeneity and the Weyl tenor for perfect fluid. This is an important expression for the Penrose proposal about the gravitational arrow of time[34]. The tidal forces associated with the Weyl tensor make the fluid more inhomogeneous as the evolution occurs, indicating the sense of time. We would like to mention here that such a relationship is no longer valid if $f(G)$ is not a constant, like local anisotropy of the pressure, dissipation and electric charge. In this case, we see from Eq.(41) how $f(G)$ affects the link between the Weyl tensor and energy density inhomogeneity, suggesting that $f(G)$ should enter into the definition of gravitational arrow of time.

5   Summary

In this paper, we have considered a new framework in which an arbitrary function $f(G)$ of Gauss-Bonnet invariant is introduced in the Einstein-Hilbert action to account for the dynamics of the shearfree dissipative gravitational collapse. The corresponding field equations are used to study the shearfree dissipative interior geometry of the star. The matching of the interior to the exterior Vaidya geometry implies that the effective pressure is balanced by the effective radial heat flux on the boundary. Using the Misner definition, we have formulated the dynamical equation (33) which shows that the Gauss-Bonnet correction increases (decreases) the inertial mass leading to increase (decrease) the rate of collapse.

To discuss the transportation of heat during the shearfree collapse of radiative fluid, we have formulated heat transport equation. Also, we have coupled the dynamical equation with the heat transport equation to study the effects of radial heat flux on the dynamics of collapsing fluid sphere. From this coupled equation (37), we find that the Gauss-Bonnet term decreases

(increases) the value of α for which F_{grav} has larger (smaller) values which increases the collapse of the system. Also, the equivalence principle is valid in $f(G)$ gravity. We have explored that when α exceeds the critical value, the gravitational force plays the role of antigravity and reversal of collapse would occur in the system. A model with bouncing behavior has been presented numerically by Herrera et al.[35] We would like to mention here that the homogeneity in energy density and conformal flatness for any metric are necessary and sufficient conditions for each other, when $f(G)$ is constant and system under consideration is composed of isotropic perfect fluid. This analysis can be extended for generalized $f(G)$ model[36] like $f(G) = C_1 G + C_2 G^{\frac{1+\alpha}{4\alpha^2}}$, C_1, C_2 and α are constants.

Appendix

The interior metric (12) has the following non-zero components of the Riemann tensor

$$
R_{0101} = XX'' + Y\ddot{Y} - \frac{X}{Y}X'Y' + \frac{Y}{X}\dot{X}\dot{Y},
$$

$$
R_{0202} = (Yr)^2 \left(-\frac{\ddot{Y}}{Y} + \frac{\dot{X}\dot{Y}}{XY} + \left(\frac{X}{Y}\right)^2 \frac{X'}{X}(\frac{Y'}{Y} + \frac{1}{r}) \right),
$$

$$
R_{0212} = (Yr)^2 \left(-\frac{\dot{Y}'}{Y} + \frac{\dot{Y}X'}{YX} + \frac{X'\dot{Y}}{YX} \right),
$$

$$
R_{1212} = (Yr)^2 \left(\left(\frac{Y}{X}\right)^2 \frac{\dot{Y}^2}{Y^2} - \frac{Y''}{Y} + \frac{Y'^2}{Y^2} - \frac{Y'}{rY} \right),
$$

$$
R_{2323} = (Yr)^2 \sin^2\theta \left(\left(\frac{r\dot{Y}}{Y}\right)^2 - \frac{r\dot{Y}'^2}{Y^2} - 2\frac{YrY'}{Y^2} \right),
$$

$$
R_{0303} = \sin^2\theta R_{0202}, \quad R_{0313} = \sin^2\theta R_{0212}, \quad R_{1313} = \sin^2\theta R_{1212}. \quad (43)
$$

This implies that the Riemann tensor has five independent components. The Kretchman scalar becomes

$$
\begin{aligned}
\mathcal{R} = & \ 4 \left(\frac{1}{(XY)^4}(R_{0101})^2 + \frac{2}{(XrY)^4}(R_{0202})^2 - \frac{4}{(XY^3r^2)^2}(R_{0212})^2 \right. \\
& + \left. \frac{2}{(Y^2r)^4}(R_{1212})^2 + \frac{1}{(Yr)^8 \sin^2\theta}(R_{2323})^2 \right). \quad (44)
\end{aligned}
$$

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The Riemann tensor components in terms of the Einstein tensor and mass function can be written as

$$
\begin{aligned}
R_{0101} &= (XY)^2 \left(\frac{1}{2X^2}G_{00} - \frac{1}{2Y^2}G_{11} + \frac{1}{(Yr)^2}G_{22} - \frac{2m}{(Yr)^3} \right), \\
R_{0202} &= (XYr)^2 \left(\frac{1}{2Y^2}G_{11} + \frac{m}{(Yr)^3} \right), \\
R_{0212} &= \frac{(Yr)^2}{2}G_{01}, \\
R_{1212} &= (Y^2 r)^2 \left(\frac{1}{2X^2}G_{00} - \frac{m}{(Yr)^3} \right), \\
R_{2323} &= 2mYr\sin^2\theta.
\end{aligned}
\tag{45}
$$

Inserting these in the Kretchman scalar (44), it follows that

$$
\begin{aligned}
\mathcal{R} &= \frac{48m^2}{(Yr)^6} - \frac{16m}{(Yr)^3} \left(\frac{G_{00}}{X^2} - \frac{G_{11}}{Y^2} + \frac{G_{22}}{(rY)^2} \right) \\
&\quad - 4\left(\frac{G_{01}}{XY}\right)^2 + 3\left(\left(\frac{G_{00}}{X^2}\right)^2 + \left(\frac{G_{11}}{Y^2}\right)^2 \right) + 4\left(\frac{G_{22}}{(rY)^2}\right)^2 \\
&\quad - 2\frac{G_{00}G_{11}}{X^2 Y^2} + 4\left(\frac{G_{00}}{X^2} - \frac{G_{11}}{Y^2} \right) \frac{G_{11}}{(rY)^2}.
\end{aligned}
\tag{46}
$$

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